Preprint typeset in JHEP style - HYPER VERSION arXiv:0711.4219v2

arXiv:0711.4219v2 [hep-th] 6 Dec 2007

Fundamental String and D1-brane in I-brane Background

by J. Klusoň

Department of Theoretical Physics and Astrophysics
Faculty of Science, Masaryk University
Kotlářská 2, 611 37, Brno
Czech Republic
E-mail: `klu@physics.muni.cz`

Abstract: This paper is devoted to the study of dynamics of fundamental string and D1-brane in I-brane background. We consider configurations where string and D1-brane uniformly wrap transverse spheres. We explicitly determine corresponding conserved charges and find relations between them.

Keywords: Fundamental Strings, D-branes.

Contents

1. Introduction and Summary

Study of different configurations of branes provides us information about their dynamics and properties of gauge theories that reside in them. One such an interesting configuration of space filling intersecting D-branes are D5-branes intersecting over $1+1$ dimensions [1]. The characteristic property of such a configurations is that they describe chiral theories in intersection domain. Further, as was observed in [2] there is an interesting symmetry enhancement where the Poincare symmetry is enhancements from $SO(1,1)$ to $SO(1,2)$ and the number of supersymmetries is doubled [1]. In fact, using the standard ideas of holography one could expect that the dynamics at the $1+1$ dimensional intersection of the two sets of fivebranes should be holographically related to a $2+1$ dimensional bulk theory, with the extra dimension being the radial direction away from the intersection. However as was nicely shown in [2] the bulk description includes two radial directions away from each set of fivebranes, and is $3+1$ dimensional and hence the corresponding boundary theory is $2+1$ dimensional. More precisely, the strong coupling limit of the system is described as a stack of intersecting NS5-branes. When all the fivebranes are coincident, the near-horizon geometry is

$$R^{2,1} \times R_\phi \times SU(2)_{k_1} \times SU(2)_{k_2} \ . \qquad (1.1)$$

Here, R_ϕ is one combination of the radial directions away from the two sets of fivebranes, and the coordinates of $R^{2,1}$ are x^0, x^1 and another combination of the two radial directions. The two $SU(2)$s describe the angular three-spheres corresponding to $(R^4)_{2345}$ and $(R^4)_{6789}$. The fact that (1.1) is an exact solution of the classical string theory equations of motion allows us to obtain information about the intersecting fivebrane system, which is not accessible via a gauge theory analysis.

As it is clear from (1.1) the geometry (1.1) exhibits a higher symmetry than the full brane configuration. In particular, the combination of radial directions away from the intersection that enters $R^{2,1}$ appears symmetrically with the other spatial direction, and

[1] For some relevant works, see [3, 4, 5, 6, 7, 8, 9, 10, 11, 12] .

the background has a higher Poincare symmetry, $ISO(2,1)$, than the expected $ISO(1,1)$ [2].

This background geometry has many unusual properties and certainly deserves better understanding. The holographic mapping between field theory living on the I-brane intersection and the bulk theory were studied in [2]. On the other hand it is also well known from the study of AdS/CFT correspondence that it is possible to derive information about the boundary CFT theory from the study of the semiclassical string and D1-brane configurations in the bulk of AdS_5 [3].

These recent developments give us the motivation to investigate semiclassical strings in I-brane background as well. In this paper we would like to investigate multispin rotating string and D1-brane solutions, having angular momenta in both three-spheres of I-brane background. Our goal is to find relation between energy and corresponding conserved charges. Unfortunately due to the limited amount of information about dual world-volume I-brane theory we will not be able to compare them with their dual counterparts. On the other hand we mean that it is useful exercise to study the dynamics of fundamental string and D1-brane in such a non-trivial background.

In fact, it turns out that the presence of a non-trivial NS two form field has an important consequences for the dynamics of string or D-brane. Explicitly, in order to find relation between energy and conserved charges we have to consider situation when string moves on both spheres S^3 simultaneously. Then we find that the dependence of energy on the conserved charges takes similar form as an ordinary relativistic relation between energy and momenta with the exception that the conserved charges related to the motion on the second three-sphere are functions of the charges related to the motion on the first three sphere.

As the next step we consider the motion of probe D1-brane in I-brane background. Study of homogeneous configurations of probe Dp-brane in similar background has been performed in the past in many papers [16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 32, 30, 31] [4] . Our approach can be considered as generalization of this approach when we study D1-brane with spatial dependent world-sheet modes. We solve equations of motion and we find explicit time dependence of the radial mode in given background. Surprisingly if we impose the condition that D1-brane radial velocity is zero we derive conditions that have the same form as Virasoro conditions in case of the fundamental string. Then we will be able to find relation between energy and conserved momenta that again has similar structure as the relation that was derived in case of fundamental string. It would be certainly interesting to study dual theory living on I-brane and try to find corresponding states that are dual to the classical fundamental string and D1-brane configurations.

As the possible extension of our work we suggest to study giant magnon and spike configurations on fundamental string in I-brane background following recent works [34, 35, 36, 37]. We hope to return to this problem in future.

[2] An important issue is whether this higher symmetry is an exact property of string theory in the background (1.1), or whether it is broken by quantum effects. This issue was carefully discussed in [2].

[3] For review, see [13, 14, 15].

[4] For review and extensive list of references, see [33].

The organization of this work is as follows. In the next section (2) we review properties of I-brane background. In section (3) we study the classical string in given background and we find solutions of the equation of motions. In section (4) we study D1-brane probe in given background and we find exact solutions of the equation of motion with some interesting properties.

2. Review of I-brane Background

In this section we review the background studied in work [2]. This background is known as I-brane background and arises from the configurations of intersecting NS5-branes. Namely, we consider the intersection of two stack of NS5-branes. We have k_1 NS5-branes extended in $(0,1,2,3,4,5)$ directions and the set of k_2 NS5-branes extended in $(0,1,6,7,8,9)$ directions. Let us define

$$\begin{aligned} \mathbf{y} &= (x^2, x^3, x^4, x^5) \ , \\ \mathbf{z} &= (x^6, x^7, x^8, x^9) \end{aligned} \tag{2.1}$$

and presume that we have k_1 NS5-branes localized at the points $\mathbf{z} = 0$ and k_2 NS5-branes localized at the points $\mathbf{y} = 0$. The supergravity background corresponding to this configuration takes the form

$$\begin{aligned} \Phi(\mathbf{z},\mathbf{y}) &= \Phi_1(\mathbf{z}) + \Phi_2(\mathbf{y}) \ , \\ g_{\mu\nu} &= \eta_{\mu\nu} \ , \quad \mu,\nu = 0,1 \ , \\ g_{\alpha\beta} &= e^{2(\Phi_2 - \Phi_2(\infty))}\delta_{\alpha\beta} \ , \quad \mathcal{H}_{\alpha\beta\gamma} = -\epsilon_{\alpha\beta\gamma\delta}\partial^\delta \Phi_2 \ , \quad \alpha,\beta,\gamma,\delta = 2,3,4,5 \ , \\ g_{pq} &= e^{2(\Phi_1 - \Phi_1(\infty))}\delta_{pq} \ , \quad \mathcal{H}_{pqr} = -\epsilon_{pqrs}\partial^s \Phi_1 \ , \quad p,q,r,s = 6,7,8,9 \ , \end{aligned} \tag{2.2}$$

where Φ on the first line means the dilaton and where

$$\begin{aligned} e^{2(\Phi_1 - \Phi_1(\infty))} &= H_1(\mathbf{z}) \equiv \frac{\lambda_1}{r_1^2} \ , \quad \lambda_1 = k_1 l_s^2 \ , \\ e^{2(\Phi_2 - \Phi_2(\infty))} &= H_2(\mathbf{y}) \equiv \frac{\lambda_2}{r_2^2} \ , \quad \lambda_2 = k_2 l_s^2 \ , \end{aligned} \tag{2.3}$$

where we consider the near horizon limit of given background. Then the metric takes the form

$$ds^2 = -dt^2 + dx_1^2 + \frac{\lambda_1}{r_1^2}dr_1^2 + \frac{\lambda_2}{r_2^2}dr_2^2 + \lambda_1 d\Omega_1^{(3)} + \lambda_2 d\Omega_2^{(3)} \ , \tag{2.4}$$

where $d\Omega_1^{(3)}$ and $d\Omega_2^{(3)}$ correspond to the line elements on the unit three sphere in the form

$$d\Omega_1^{(3)} = d\theta_1^2 + \sin^2\theta_1 d\phi_1^2 + \cos^2\theta_1 d\psi_1^2 \ ,$$

$$
\begin{aligned}
0 < \theta_1 < \frac{\pi}{2} \ , \quad 0 = \phi_1 \ , \quad \psi_1 < 2\pi \ , \\
d\Omega_2^{(3)} = d\theta_2^2 + \sin^2\theta_2 d\phi_2^2 + \cos^2\theta_2 d\psi_2^2 \ , \\
0 < \theta_2 < \frac{\pi}{2} \ , \quad 0 = \phi_2 \ , \quad \psi_2 < 2\pi \ .
\end{aligned}
\tag{2.5}
$$

Finally, we choose the gauge where the non-zero components of NS two form take the form

$$
b^{(1)}_{\phi_1\psi_1} = \lambda_1 \cos^2\theta_1 \ , \quad b^{(2)}_{\phi_2\psi_2} = \lambda_2 \cos^2\theta_2 \ .
\tag{2.6}
$$

3. Fundamental String in I-brane Background

In this section we study dynamics of fundamental string in I-brane background. Our starting point is the Polyakov form of the string action in general background

$$
\begin{aligned}
S = & -\frac{1}{4\pi\alpha'}\int_{-\pi}^{\pi} d\sigma d\tau [\sqrt{-\gamma}\gamma^{\alpha\beta} g_{MN}\partial_\alpha x^M \partial_\beta x^N - e^{\alpha\beta}\partial_\alpha x^M \partial_\beta x^N b_{MN}] + \\
& + \frac{1}{4\pi}\int_{-\pi}^{\pi} d\sigma d\tau \sqrt{-\gamma} R\Phi \ ,
\end{aligned}
\tag{3.1}
$$

where $\gamma^{\alpha\beta}$ is a world-sheet metric, R is its Ricci scalar and $e^{\tau\sigma} = -\epsilon^{\sigma\tau} = 1$. Finally, the modes x^M , $M = 0, \dots, 9$ parameterize the embedding of the string in given background. The variation of the action (3.1) with respect to x^K implies following equations of motion

$$
\begin{aligned}
& -\frac{1}{4\pi\alpha'}\sqrt{-\gamma}\gamma^{\alpha\beta}\partial_K g_{MN}\partial_\alpha x^M \partial_\beta x^N + \frac{1}{2\pi\alpha'}\partial_\alpha[\sqrt{-\gamma}\gamma^{\alpha\beta} g_{KM}\partial_\beta x^M] - \\
& - \frac{1}{2\pi\alpha'}\partial_\alpha[\epsilon^{\alpha\beta}\partial_\beta x^M b_{KM}] + \frac{1}{4\pi\alpha'}\epsilon^{\alpha\beta}\partial_\alpha x^M \partial_\beta x^N \partial_K b_{MN} + \frac{1}{4\pi}\partial_K\Phi\sqrt{-\gamma}R = 0 \ .
\end{aligned}
\tag{3.2}
$$

Further, the variation of the action with respect to the metric components imply the constraints

$$
\begin{aligned}
T_{\alpha\beta} \equiv & -\frac{4\pi}{\sqrt{-\gamma}}\frac{\delta S}{\delta\gamma^{\alpha\beta}} = \frac{1}{\alpha'} g_{MN}\partial_\alpha x^M \partial_\beta x^N - R_{\alpha\beta} + \\
& + (\nabla_\alpha\nabla_\beta x^M)\partial_M\Phi + (\partial_\alpha x^M \partial_\beta x^N)\partial_M\partial_N\Phi - \\
& - \frac{1}{2}\gamma_{\alpha\beta}\left(\frac{1}{\alpha'}\gamma^{\gamma\delta}\partial_\gamma x^M \partial_\delta x^N g_{MN} - R\Phi + 2\nabla^\alpha\nabla_\alpha\Phi\right) \ .
\end{aligned}
\tag{3.3}
$$

As the first step we introduce two modes ρ_1 and ρ_2 defined as

$$
r_1 = e^{\frac{\rho_1}{\sqrt{\lambda_1}}} \ , \quad r_2 = e^{\frac{\rho_2}{\sqrt{\lambda_2}}} \ .
\tag{3.4}
$$

In the second step, following [2] we introduce two modes r, y through the relation

$$Qr = \frac{1}{\sqrt{\lambda_1}}\rho_1 + \frac{1}{\sqrt{\lambda_2}}\rho_2 \ , \quad Qy = \frac{1}{\sqrt{\lambda_2}}\rho_1 - \frac{1}{\sqrt{\lambda_1}}\rho_2 \ , \tag{3.5}$$

where

$$Q = \frac{1}{\sqrt{\lambda}} \ , \quad \frac{1}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} \ . \tag{3.6}$$

Using these transformations it is easy to see that the dilaton Φ depends on r only

$$\Phi = \Phi_1 + \Phi_2 = -Qr + \Phi_0 \ , \tag{3.7}$$

where $\Phi_0 \equiv \Phi_1(\infty) + \Phi_2(\infty)$. With the help of the variables r, y the action for string in I-brane background takes the form

$$\begin{aligned} S = & -\frac{1}{4\pi\alpha'}\int_{-\pi}^{\pi} d\sigma d\tau [\sqrt{-\gamma}\gamma^{\alpha\beta}(-\partial_\alpha t\partial_\beta t + \partial_\alpha r\partial_\beta r + \partial_\alpha y\partial_\beta y + \\ & + g_{mn}\partial_\alpha x^m\partial_\beta x^n) - e^{\alpha\beta}\partial_\alpha x^m\partial_\beta x^n b_{mn}] - \\ & - \frac{1}{4\pi}\int_{-\pi}^{\pi} d\sigma d\tau\sqrt{-\gamma}RQr \ , \end{aligned} \tag{3.8}$$

where x^m label angular coordinates corresponding to S^3_1, S^3_2 respectively. In the conformal gauge $\gamma_{\alpha\beta} = \eta_{\alpha\beta}$ the constraints (3.3) that follow from the variation of the action (3.8) take simpler form

$$\begin{aligned} T_{\sigma\sigma} &= \frac{1}{2\alpha'}(-\partial_\sigma t\partial_\sigma t - \partial_\tau t\partial_\tau t + \partial_\sigma r\partial_\sigma r + \partial_\tau r\partial_\tau r + \partial_\sigma y\partial_\sigma y + \partial_\tau y\partial_\tau y + \\ & + g_{mn}\partial_\sigma x^m\partial_\sigma x^n + g_{mn}\partial_\tau x^m\partial_\tau x^n) - Q\partial_\tau^2 r \ , \\ T_{\tau\tau} &= \frac{1}{2\alpha'}(-\partial_\sigma t\partial_\sigma t - \partial_\tau t\partial_\tau t + \partial_\sigma r\partial_\sigma r + \partial_\tau r\partial_\tau r + \partial_\sigma y\partial_\sigma y + \partial_\tau y\partial_\tau y + \\ & + g_{mn}\partial_\sigma x^m\partial_\sigma x^n + g_{mn}\partial_\tau x^m\partial_\tau x^n) - Q\partial_\sigma^2 r \ , \\ T_{\tau\sigma} &= \frac{1}{\alpha'}(-\partial_\tau t\partial_\sigma t + \partial_\tau r\partial_\sigma r + \partial_\tau y\partial_\sigma y + g_{mn}\partial_\sigma x^m\partial_\tau x^n) - Q\partial_\sigma\partial_\tau r \ . \end{aligned} \tag{3.9}$$

Looking on the form of the background (2.4) we observe that the action (3.8) is invariant under following transformations of fields

$$\begin{aligned} t'(\tau,\sigma) &= t(\sigma,\tau) + \epsilon_t \ , \\ y'(\tau,\sigma) &= y(\tau,\sigma) + \epsilon_y \ , \\ \psi_1'(\tau,\sigma) &= \psi_1(\tau,\sigma) + \epsilon_{\psi_1} \ , \\ \psi_2'(\tau,\sigma) &= \psi_2(\tau,\sigma) + \epsilon_{\psi_2} \ , \\ \phi_1'(\tau,\sigma) &= \phi_1(\tau,\sigma) + \epsilon_{\phi_1} \ , \\ \phi_2'(\tau,\sigma) &= \phi_2(\tau,\sigma) + \epsilon_{\phi_2} \ , \end{aligned} \tag{3.10}$$

where $\epsilon_t, \epsilon_y, \epsilon_{\phi_{1,2}}, \epsilon_{\psi_{1,2}}$ are constants. Then it is simple task to determine corresponding conserved charges

$$\begin{aligned}
P_t &= \frac{1}{2\pi\alpha'}\int_{-\pi}^{\pi} d\sigma \partial_\tau t \ , \quad P_y = -\frac{1}{2\pi\alpha'}\int_{-\pi}^{\pi} d\sigma \partial_\alpha y \ , \\
P_{\psi_1} &= -\frac{1}{2\pi\alpha'}\int_{-\pi}^{\pi} d\sigma [g_{\psi_1\psi_1}\partial_\tau\psi_1 + b_{\psi_1\phi_1}\partial_\sigma\phi_1] \ , \\
P_{\psi_2} &= -\frac{1}{2\pi\alpha'}\int_{-\pi}^{\pi} d\sigma [g_{\psi_2\psi_2}\partial_\tau\psi_2 + b_{\psi_2\phi_2}\partial_\sigma\phi_2] \ , \\
P_{\phi_1} &= -\frac{1}{2\pi\alpha'}\int_{-\pi}^{\pi} d\sigma [g_{\phi_1\phi_1}\partial_\tau\phi_1 + b_{\phi_1\psi_1}\partial_\sigma\psi_1] \ , \\
P_{\phi_2} &= -\frac{1}{2\pi\alpha'}\int_{-\pi}^{\pi} d\sigma [g_{\phi_2\phi_2}\partial_\tau\phi_2 + b_{\phi_2\psi_2}\partial_\sigma\psi_2] \ .
\end{aligned} \tag{3.11}$$

Note that P_t is related to the energy as $P_t = -E$.

Our goal is to study the dynamics of string that moves and wraps two spheres $S^3_{1,2}$ simultaneously. Explicitly, let us consider following ansatz

$$\begin{aligned}
r &= r(\tau), \quad y = y(\tau), \quad t = \kappa\tau, \\
\theta_1 &= \theta_1^c \equiv \text{const} \ , \quad \psi_1 = \omega_1\tau + n_1\sigma \ , \quad \phi_1 = \nu_1\tau + m_1\sigma \ , \\
\theta_2 &= \theta_2^c \equiv \text{const}, \quad \psi_2 = \omega_2\tau + n_2\sigma \ , \quad \phi_2 = \nu_2\tau + m_2\sigma \ .
\end{aligned} \tag{3.12}$$

Let us now study the equations of motion (3.2) for string moving in the background (2.4). In conformal gauge the equations of motion for t, y, r take the form

$$\partial_\alpha[\eta^{\alpha\beta}\partial_\beta t] = 0 \ , \quad \partial_\alpha[\eta^{\alpha\beta}\partial_\beta r] = 0 \ , \quad \partial_\alpha[\eta^{\alpha\beta}\partial_\beta y] = 0 \tag{3.13}$$

that we solve as

$$t = \kappa\tau \ , \quad r = v_r\tau + r_0 \ , \quad y = v_y\tau + y_0 \ . \tag{3.14}$$

Further, the equations of motion for θ_1 and θ_2 for constant θ's take the form

$$\frac{\lambda_1 \sin\theta_1^c \cos\theta_1^c}{2\pi\alpha'}[\nu_1^2 - m_1^2 - \omega_1^2 + n_1^2 + 2\omega_1 m_1 - 2\nu_1 n_1] = 0 \ , \tag{3.15}$$

$$\frac{\lambda_2 \sin\theta_2^c \cos\theta_2^c}{2\pi\alpha'}[\nu_2^2 - m_2^2 - \omega_2^2 + n_2^2 + 2\omega_2 m_2 - 2\nu_2 n_2] = 0 \ . \tag{3.16}$$

Finally, the equation of motion for ϕ_1 in I-brane background takes the form

$$\frac{1}{2\pi\alpha'}\partial_\alpha[\eta^{\alpha\beta}g_{\phi_1\phi_1}\partial_\beta\phi_1] - \frac{1}{2\pi\alpha'}\partial_\alpha[\epsilon^{\alpha\beta}\partial_\beta\psi_1 b_{\phi_1\psi_1}] = 0 \tag{3.17}$$

that is automatically obeyed for (3.12). In the same way we can show that the equations of motion for ψ_1, ϕ_2, ψ_2 are obeyed with the ansatz (3.12). Further, it is easy to see that for

$$\begin{aligned}\omega_1 &= \nu_1 \ , \quad m_1 = n_1 \ , \\ \omega_2 &= \nu_2 \ , \quad m_2 = n_2\end{aligned} \tag{3.18}$$

the equations of motion (3.15), (3.16) are satisfied as well.

As the next step we are going to analyze Virasoro constraints (3.9). The constraint $T_{\tau\sigma} = 0$ implies

$$\lambda_1 \sin^2 \theta_1^c \nu_1 m_1 + \lambda_1 \cos^2 \theta_1^c \omega_1 n_1 + \lambda_2 \sin^2 \theta_2^c \nu_2 m_2 + \lambda_2 \cos^2 \theta_2^c \omega_2 n_2 = 0 \ . \tag{3.19}$$

If we insert (3.18) into (3.19) we obtain the condition

$$\frac{\lambda_1}{\lambda_2} = -\frac{\omega_2 m_2}{\omega_1 m_1} \tag{3.20}$$

that can be solved as

$$m_1 = -m_2 \ , \quad \omega_2 = \frac{\lambda_1}{\lambda_2}\omega_1 \ . \tag{3.21}$$

Note that this solution is valid for any constant $\theta_{1,2}^c$. On the other hand the Virasoro constraints $T_{\tau\tau} = T_{\sigma\sigma} = 0$ imply

$$-\kappa^2 + v_r^2 + v_y^2 + \lambda_1(\omega_1^2 + m_1^2) + \lambda_2(\omega_2^2 + m_2^2) = 0 \ . \tag{3.22}$$

Let us now determine the form of the conserved charges $P_t, P_y, P_{\phi_1}, P_{\phi_2}, P_{\psi_1}$ and P_{ψ_2} for the ansatz (3.12)

$$\begin{aligned}P_t &= \frac{1}{\alpha'}\kappa \ , \quad P_y = -\frac{1}{\alpha'}v_y \ , \\ P_{\psi_1} &= \frac{\lambda_1}{\alpha'}[-\cos^2 \theta_1^c \omega_1 + \cos^2 \theta_1^c m_1] \ , \\ P_{\psi_2} &= \frac{\lambda_2}{\alpha'}[-\cos^2 \theta_2^c \omega_2 + \cos^2 \theta_2^c m_2] \ , \\ P_{\phi_1} &= \frac{\lambda_1}{\alpha'}[-\sin^2 \theta_1^c \nu_1 - \cos^2 \theta_1^c n_1] \ , \\ P_{\phi_2} &= \frac{\lambda_2}{\alpha'}[-\sin^2 \theta_2^c \nu_1 - \cos^2 \theta_2^c n_2] \ .\end{aligned} \tag{3.23}$$

Inverting these relations we can express ω_1, ω_2, m_1 and m_2 as functions of conserved charges

$$\begin{aligned}\omega_1 &= -\frac{\alpha'}{\lambda_1}[P_{\psi_1} + P_{\phi_1}] \ , \quad m_1 = \frac{\alpha'}{\lambda_1 \cos^2 \theta_1^c}[P_{\psi_1} \sin^2 \theta_1^c - P_{\phi_1} \cos^2 \theta_1^c] \ , \\ \omega_2 &= -\frac{\alpha'}{\lambda_1}[P_{\psi_2} + P_{\phi_2}] \ , \quad m_2 = \frac{\alpha'}{\lambda_2 \cos^2 \theta_2^c}[P_{\psi_2} \sin^2 \theta_2^c - P_{\phi_2} \cos^2 \theta_2^c] \ .\end{aligned} \tag{3.24}$$

Then using (3.22) we find the dependence of energy on the conserved charges in the form

$$
\begin{aligned}
E^2 &= P_r^2 + P_y^2 + \frac{1}{\lambda_1}(P_{\psi_1} + P_{\phi_1})^2 + \frac{1}{\lambda_2}(P_{\psi_2} + P_{\phi_2})^2 + \\
&+ \frac{\lambda_1 (2\pi m_1)^2}{(2\pi\alpha')^2} + \frac{\lambda_2 (2\pi m_2)^2}{(2\pi\alpha')^2} \ ,
\end{aligned}
\tag{3.25}
$$

where $m_1 = -m_2$, P_r is radial momentum [5] and where P_{ϕ_2}, P_{ψ_2} are related to P_{ϕ_1} and P_{ψ_1} as

$$
\begin{aligned}
P_{\phi_2} &= \frac{\lambda_2}{\lambda_1} P_{\phi_1} (\sin^2\theta_2^c - \cos^2\theta_2^c) + P_{\psi_1} \frac{\lambda_2}{\lambda_1} \left(\frac{\sin^2\theta_2^c \cos^2\theta_1^c + \cos^2\theta_2^c \sin^2\theta_1^c}{\cos^2\theta_1} \right) \\
P_{\psi_2} &= 2\frac{\lambda_2}{\lambda_1} P_{\phi_1} \cos^2\theta_2^c + \frac{\lambda_2}{\lambda_1} P_{\psi_1} \frac{\cos^2\theta_2^c}{\cos^2\theta_1^c} \left(\cos^2\theta_1^c - \sin^2\theta_1^c\right) \ .
\end{aligned}
\tag{3.26}
$$

We see that in spite of the fact that angular momenta are related through the relations (3.26) the dispersion relation between energy and conserved charges takes rather simple form. Then it would be really interesting to identify corresponding states in dual I-brane world-volume theory.

4. D1-brane in I-brane Background

In this section we generalize discussion presented in previous section to the case of D1-brane that moves in I-brane background. Recall that the dynamics of probe D1-brane in general background is governed by the action

$$
\begin{aligned}
S &= S_{DBI} + S_{WZ} \ , \\
S_{DBI} &= -\tau_1 \int d^2\xi e^{-\Phi} \sqrt{-\det \mathbf{A}} \ , \\
\mathbf{A}_{\alpha\beta} &= \partial_\alpha x^M \partial_\beta x^N g_{MN} + (2\pi\alpha') \mathcal{F}_{\alpha\beta} \ , \\
\mathcal{F}_{\alpha\beta} &= \partial_\alpha A_\beta - \partial_\beta A_\alpha - (2\pi\alpha')^{-1} b_{MN} \partial_\alpha x^M \partial_\beta x^N \ ,
\end{aligned}
\tag{4.1}
$$

where $\tau_1 = \frac{1}{2\pi\alpha'}$ is D1-brane tension, $\xi^\alpha, \alpha = 0, 1, \xi^0 \equiv \tau, \xi^1 = \sigma$ are world-volume coordinates and where A_α is gauge field living on the world-volume of D1-brane.

If we now perform the variation of (4.1) with respect to x^M we obtain following equations of motion for x^M

$$
\begin{aligned}
& -\tau_1 \partial_M [e^{-\Phi}] \sqrt{-\det \mathbf{A}} \\
& - \frac{\tau_1}{2} e^{-\Phi} (\partial_M g_{KL} \partial_\alpha x^K \partial_\beta x^L - \partial_M b_{KL} \partial_\alpha x^K \partial_\beta x^L) \left(\mathbf{A}^{-1}\right)^{\beta\alpha} \sqrt{-\det \mathbf{A}} + \\
& + \tau_1 \partial_\alpha [e^{-\Phi} g_{MN} \partial_\beta x^N \left(\mathbf{A}^{-1}\right)_S^{\beta\alpha} \sqrt{-\det \mathbf{A}}] - \\
& - \tau_1 \partial_\alpha [e^{-\Phi} b_{MN} \partial_\beta x^N \left(\mathbf{A}^{-1}\right)_A^{\beta\alpha} \sqrt{-\det \mathbf{A}}] = 0 \ .
\end{aligned}
\tag{4.2}
$$

[5] Note that the motion of the classical string along radial direction in I-brane background is free when we impose conformal gauge.

In the same way the variation of (4.1) with respect to A_α implies following equation of motion

$$\partial_\alpha[e^{-\Phi}\left(\mathbf{A}^{-1}\right)^{\beta\alpha}_A\sqrt{-\det\mathbf{A}}]=0\ . \tag{4.3}$$

Now we concentrate on dynamics of D1-brane in I-brane background and consider ansatz

$$\begin{gathered}
t=\kappa\tau\ ,\quad r=r(\tau)\ ,\quad y=y(\tau)\ ,\\
\psi_1=\omega_1\tau+n_1\sigma\ ,\quad \phi_1=\nu_1\tau+m_1\sigma\ ,\\
\psi_2=\omega_2\tau+n_2\sigma\ ,\quad \phi_2=\nu_2\tau+m_2\sigma
\end{gathered} \tag{4.4}$$

and hence the matrix elements $\mathbf{A}_{\alpha\beta}$ take the form

$$\begin{aligned}
\mathbf{A}_{\tau\tau} &= -\kappa^2+\dot{r}^2+\dot{y}^2+\lambda_1\sin^2\theta_1\nu_1^2+\lambda_1\cos^2\theta_1\omega_1^2+\lambda_2\sin^2\theta_2\nu_2^2+\lambda_2\cos^2\theta_2\omega_2^2\\
\mathbf{A}_{\sigma\sigma} &= \lambda_1\sin^2\theta_1 m_1^2+\lambda_2\cos^2\theta_1 n_1^2+\lambda_2\sin^2\theta_2 m_2^2+\lambda_2\cos^2\theta_2 n_2^2\ ,\\
\mathbf{A}_{\tau\sigma} &= \lambda_1\sin^2\theta_1\nu_1 m_1+\lambda_1\cos^2\theta_1\omega_1 n_1+\lambda_2\sin^2\theta_2\nu_2 m_2+\lambda_2\cos^2\theta_2\omega_2 n_2-\\
&-\lambda_1\cos^2\theta_1(\nu_1 n_1-\omega_1 m_1)-\lambda_2\cos^2\theta_2(\nu_2 n_2-\omega_2 m_2)+(2\pi\alpha')F\ ,\\
\mathbf{A}_{\sigma\tau} &= \lambda_1\sin^2\theta_1\nu_1 m_1+\lambda_1\cos^2\theta_1\omega_1 n_1+\lambda_2\sin^2\theta_2\nu_2 m_2+\lambda_2\cos^2\theta_2\omega_2 n_2+\\
&+\lambda_1\cos^2\theta_1(\nu_1 n_1-\omega_1 m_1)+\lambda_2\cos^2\theta_2(\nu_2 n_2-\omega_2 m_2)-(2\pi\alpha')F\ ,
\end{aligned} \tag{4.5}$$

where $F\equiv F_{\tau\sigma}$, $\dot{r}=\partial_\tau r$, $\dot{y}=\partial_\tau y$. Again, it is easy to see that the time dependence of y is $y(\tau)=y_0+v_y\tau$.

As the next step we use the fact that (4.3) implies an existence of conserved flux Π

$$\frac{e^{-\Phi}(\mathbf{A}_{\tau\sigma})^A}{\sqrt{-\det\mathbf{A}}}=\Pi\ . \tag{4.6}$$

Then, after some algebra, we obtain

$$\begin{aligned}
\sqrt{-\det\mathbf{A}} &= \frac{1}{\sqrt{1+e^{-2Qr}g_s^2\Pi^2}}[-\dot{r}^2\mathbf{A}_{\sigma\sigma}+\mathbf{M}]\ ,\\
\mathbf{M} &= -\mathbf{A}'_{\tau\tau}\mathbf{A}_{\sigma\sigma}+(\mathbf{A}_{\tau\sigma})^S(\mathbf{A}_{\tau\sigma})^S\ ,\quad \mathbf{A}'_{\tau\tau}=\mathbf{A}_{\tau\tau}-\dot{r}^2\ ,
\end{aligned} \tag{4.7}$$

where $\mathbf{A}'_{\tau\tau}$ and $\mathbf{M}$ are constants.

In order to find the time dependence of r let us consider the equation of motion for $x^0=t$ that for the ansatz (4.4) implies an existence of the conserved quantity

$$\frac{e^{-\Phi}\mathbf{A}_{\sigma\sigma}}{\sqrt{-\det\mathbf{A}}}=A\ ,\quad \mathrm{A}=const\ . \tag{4.8}$$

Then using (4.8) we easily determine the differential equation for r

$$\dot{r}=\sqrt{\frac{\mathbf{M}}{\mathbf{A}_{\sigma\sigma}}-\frac{\mathbf{A}_{\sigma\sigma}\Pi^2}{A^2}}\sqrt{1-\frac{\mathbf{A}_{\sigma\sigma}^2}{\mathbf{M}A^2-\mathbf{A}_{\sigma\sigma}^2\Pi^2}\frac{e^{2Qr}}{g_s^2}}\ . \tag{4.9}$$

The differential equation given above can be easily integrated and we obtain the result

$$\frac{1}{\cosh\left(\tau Q\sqrt{\frac{\mathbf{M}A^2-\mathbf{A}_{\sigma\sigma}^2\Pi^2}{\mathbf{A}_{\sigma\sigma}A^2}}\right)}=\sqrt{\frac{(\mathbf{M}A^2-\mathbf{A}_{\sigma\sigma}^2\Pi^2)g_s^2}{\mathbf{A}_{\sigma\sigma}A^2}}e^{Qr}\ , \tag{4.10}$$

where we imposed the initial condition that at time $\tau_0=0$ D1-brane is localized in its turning point where $\dot{r}=0$.

As the next step we solve the equations of motion for $\phi_{1,2},\psi_{1,2}$. For example, let us consider the ansatz (4.4) in the equation of motion for ϕ_1

$$\begin{aligned}&\partial_\alpha[e^{-\Phi}g_{\phi_1\phi_1}\partial_\beta\phi_1\left(\mathbf{A}^{-1}\right)_S^{\beta\alpha}\sqrt{-\det\mathbf{A}}]-\partial_\alpha[e^{-\Phi}b_{\phi_1\psi_1}\partial_\beta\psi_1\left(\mathbf{A}^{-1}\right)_A^{\beta\alpha}\sqrt{-\det\mathbf{A}}]=\\&=\nu_1\partial_\tau[g_{\phi_1\phi_1}\frac{e^{-\Phi}\mathbf{A}_{\sigma\sigma}}{\sqrt{-\det\mathbf{A}}}]+n_1\partial_\tau[b_{\phi_1\psi_1}\frac{e^{-\Phi}(\mathbf{A}_{\tau\sigma})^A}{\sqrt{-\det\mathbf{A}}}]=0\ ,\end{aligned} \tag{4.11}$$

where in the first step we used the fact that $g_{\phi_1\phi_1}$ is constant for constant θ_1, in the second step the fact that $\det\mathbf{A}$ is a function of τ only and in the final step (4.8) and (4.6). In the same way we can show that the equations of motion for ψ_1,ϕ_2,ψ_2 are satisfied.

Let us now analyze the equations of motion for θ_1,θ_2. For constant θ_1 its equation of motion takes the form

$$\begin{aligned}&\frac{e^{-\Phi}}{\sqrt{1+e^{2\Phi}\Pi^2}\sqrt{-\mathbf{A}_{\tau\tau}\mathbf{A}_{\sigma\sigma}+(\mathbf{A}_{\tau\sigma})^S(\mathbf{A}_{\tau\sigma})^S}}\times\\&\times\left(-\frac{\delta\mathbf{A}_{\sigma\sigma}}{\delta\theta_1}\mathbf{A}_{\tau\tau}-\mathbf{A}_{\sigma\sigma}\frac{\delta\mathbf{A}_{\tau\tau}}{\delta\theta_1}+2(\mathbf{A}_{\tau\sigma})^S\frac{\delta(\mathbf{A}_{\tau\sigma})^S}{\delta\theta_1}\right)=0\ .\end{aligned} \tag{4.12}$$

Since $\mathbf{A}_{\tau\tau}$ contains $\dot{r}$ it is manifestly time dependent. Then in order to obey (4.12) we have to demand that

$$\begin{aligned}\frac{\delta\mathbf{A}_{\sigma\sigma}}{\delta\theta_1}&=2\lambda_1\sin\theta_1\cos\theta_1(m_1^2-n_1^2)=0\ ,\\\frac{\delta\mathbf{A}_{\tau\tau}}{\delta\theta_1}&=2\lambda_1\sin\theta_1\cos\theta_1(\omega_1^2-\nu_1^2)=0\ .\end{aligned} \tag{4.13}$$

We solve these equations by imposing following relations

$$m_1=n_1\ ,\quad \omega_1=\nu_1\ . \tag{4.14}$$

Then it is easy to see that $\frac{\delta\mathbf{A}_{\tau\sigma}}{\delta\theta_1}=\lambda_1\sin\theta_1\cos\theta_1(\nu_1m_1-\omega_1n_1)$ is satisfied for (4.14) as well. In the same way we derive

$$m_2=n_2\ ,\quad \omega_2=\nu_2 \tag{4.15}$$

from the equation of motion for θ_2. Then we obtain

$$\begin{aligned}&\mathbf{M}=-\kappa^2+v_y^2+\lambda_1(\omega_1^2+n_1^2)+\lambda_2(\omega_2^2+n_2^2)\ ,\\&\mathbf{A}_{\sigma\sigma}=\lambda_1n_1^2+\lambda_2n_2^2\ ,\\&(\mathbf{A}_{\sigma\tau})^S=\lambda_1\omega_1n_1+\lambda_2\omega_2n_2\ .\end{aligned} \tag{4.16}$$

As follows from the form of the I-brane background D1-brane possesses following conserved currents

$$\begin{aligned}
\mathcal{J}^\alpha_y &= \frac{\delta\mathcal{L}}{\delta\partial_\alpha y} = -\tau_1 e^{-\Phi} g_{yy}\partial_\beta y \left(\mathbf{A}^{-1}\right)^{\beta\alpha}_S \sqrt{-\det\mathbf{A}} \ , \\
\mathcal{J}^\alpha_t &= \frac{\delta\mathcal{L}}{\delta\partial_\alpha t} = -\tau_1 e^{-\Phi} g_{tt}\partial_\beta t \left(\mathbf{A}^{-1}\right)^{\beta\alpha}_S \sqrt{-\det\mathbf{A}} \ , \\
\mathcal{J}^\alpha_{\phi_1} &= \frac{\delta\mathcal{L}}{\delta\partial_\alpha \phi_1} = -\tau_1 e^{-\Phi} [g_{\phi_1\phi_1}\partial_\beta\phi_1 \left(\mathbf{A}^{-1}\right)^{\beta\alpha}_S + b_{\phi_1\psi_1}\partial_\beta\psi_1 \left(\mathbf{A}^{-1}\right)^{\beta\alpha}_A]\sqrt{-\det\mathbf{A}} \ , \\
\mathcal{J}^\alpha_{\phi_2} &= \frac{\delta\mathcal{L}}{\delta\partial_\alpha \phi_2} = -\tau_1 e^{-\Phi} [g_{\phi_2\phi_2}\partial_\beta\phi_2 \left(\mathbf{A}^{-1}\right)^{\beta\alpha}_S + b_{\phi_2\psi_2}\partial_\beta\psi_2 \left(\mathbf{A}^{-1}\right)^{\beta\alpha}_A]\sqrt{-\det\mathbf{A}} \ , \\
\mathcal{J}^\alpha_{\psi_1} &= \frac{\delta\mathcal{L}}{\delta\partial_\alpha \psi_1} = -\tau_1 e^{-\Phi} [g_{\psi_1\psi_1}\partial_\beta\psi_1 \left(\mathbf{A}^{-1}\right)^{\beta\alpha}_S + b_{\psi_1\phi_1}\partial_\beta\phi_1 \left(\mathbf{A}^{-1}\right)^{\beta\alpha}_A]\sqrt{-\det\mathbf{A}} \ , \\
\mathcal{J}^\alpha_{\psi_2} &= \frac{\delta\mathcal{L}}{\delta\partial_\alpha \psi_2} = -\tau_1 e^{-\Phi} [g_{\psi_2\psi_2}\partial_\beta\psi_2 \left(\mathbf{A}^{-1}\right)^{\beta\alpha}_S + b_{\psi_2\phi_2}\partial_\beta\psi_2 \left(\mathbf{A}^{-1}\right)^{\beta\alpha}_A]\sqrt{-\det\mathbf{A}} \ .
\end{aligned} \tag{4.17}$$

These currents are conserved as a consequence of the equation of motion:

$$\partial_\alpha \mathcal{J}^\alpha_x = 0 \ , \quad x = (t, \phi_1, \phi_2, \psi_1, \psi_2) \ . \tag{4.18}$$

Then corresponding conserved charges take the form

$$\begin{aligned}
P_t &= \int_0^{2\pi} d\sigma \mathcal{J}^\tau_t \ , \quad P_y = \int_0^{2\pi} d\sigma \mathcal{J}^\tau_y \ , \\
P_{\phi_{1,2}} &= \int_0^{2\pi} d\sigma \mathcal{J}^\tau_{\phi_{1,2}} \ , \quad P_{\psi_{1,2}} = \int_0^{2\pi} d\sigma \mathcal{J}^\tau_{\psi_{1,2}}
\end{aligned} \tag{4.19}$$

that, for the ansatz (4.4) are equal to

$$\begin{aligned}
P_t &= -\tau_1 2\pi\kappa A \ , \quad P_y = \tau_1 2\pi v_y A \ , \\
P_{\phi_1} &= 2\pi\tau_1 \sin^2\theta_1 A \frac{\lambda_1\lambda_2(n_2\omega_1 - \omega_2 n_1)n_2}{\lambda_1 n_1^2 + \lambda_2 n_2^2} - 2\pi\tau_1\lambda_1\cos^2\theta_1 n_1 \Pi \ , \\
P_{\psi_1} &= 2\pi\tau_1 \cos^2\theta_1 A \frac{\lambda_1\lambda_2(n_2\omega_1 - \omega_2 n_1)n_2}{\lambda_1 n_1^2 + \lambda_2 n_2^2} + 2\pi\tau_1\lambda_1\cos^2\theta_1 n_1 \Pi \ , \\
P_{\phi_2} &= -2\pi\tau_1 \sin^2\theta_2 A \frac{\lambda_1\lambda_2(n_2\omega_1 - \omega_2 n_1)n_1}{\lambda_1 n_1^2 + \lambda_2 n_2^2} - 2\pi\tau_1\lambda_2\cos^2\theta_2 n_2 \Pi \ , \\
P_{\psi_2} &= -2\pi\tau_1 \cos^2\theta_2 A \frac{\lambda_1\lambda_2(n_2\omega_1 - \omega_2 n_1)n_1}{\lambda_1 n_1^2 + \lambda_2 n_2^2} + 2\pi\tau_1\lambda_1\cos^2\theta_2 n_2 \Pi \ .
\end{aligned} \tag{4.20}$$

The remarkable property of the solution given above is that there is not any relation between energy and angular momenta. Remember that in case of the fundamental string an existence of this relation follows from the Virasoro constraints. Interestingly we can

find similar relation when we impose the condition $\dot{r} = 0$. In fact, as follows from (4.9) this situation occurs when

$$A^2(-\mathbf{A}'_{\tau\tau}\mathbf{A}_{\sigma\sigma} + (\mathbf{A}_{\tau\sigma})^S(\mathbf{A}_{\tau\sigma})^S) - \mathbf{A}^2_{\sigma\sigma}\Pi^2 = 0 \ . \tag{4.21}$$

The equation above can be solved with the ansatz

$$\mathbf{A}'_{\tau\tau} = \mathbf{A}_{\tau\tau} = -\mathbf{A}_{\sigma\sigma} \ , \quad (\mathbf{A}_{\tau\sigma})^S = 0 \ , \quad \Pi^2 = A^2 \ , \tag{4.22}$$

where we used the fact that for $\dot{r}^2 = 0$, $\mathbf{A}'_{\tau\tau} = \mathbf{A}_{\tau\tau}$. Using these prescriptions we derive constraints that have similar form as Virasoro constraints for fundamental string.

Further, using (4.22) it is easy to see that the equation of motion for θ_1 is solved with the ansatz (4.14) and the equation of motion for θ_2 with the ansatz (4.15). Then the condition $(\mathbf{A}_{\tau\sigma})^S = 0$ implies

$$\lambda_1\omega_1 n_1 + \lambda_2 n_2\omega_2 = 0 \tag{4.23}$$

that can be also solved as

$$\omega_2 = \omega_1\frac{\lambda_1}{\lambda_2} \ , \quad n_1 = -n_2 \tag{4.24}$$

that has again the same form as the relations derived in previous section. Then, for (4.23) and for $A = \Pi$ we obtain

$$\begin{aligned}
P_t &= -\tau_1 2\pi\Pi\kappa \ , \quad P_y = \tau_1 2\pi\Pi v_y \ , \\
P_{\phi_1} &= 2\pi\tau_1\Pi\lambda_1[\sin^2\theta_1\omega_1 - \cos^2\theta_1 m_1] \ , \\
P_{\psi_1} &= 2\pi\tau_1\Pi\lambda_1[\cos^2\theta_1\omega_1 + \cos^2\theta_1 m_1] \ , \\
P_{\phi_2} &= 2\pi\tau_1\Pi\lambda_2[\sin^2\theta_2\omega_2 - \cos^2\theta_2 m_2] \ , \\
P_{\psi_2} &= 2\pi\tau_1\Pi\lambda_2[\sin^2\theta_2\omega_2 + \cos^2\theta_2 m_2] \ ,
\end{aligned} \tag{4.25}$$

where again ω_2, m_2 are related to ω_1, m_1 through the relations (4.24) that allow to express P_{ϕ_2}, P_{ψ_2} as functions of P_{ϕ_1}, P_{ψ_1} exactly in the same way as in previous section. Further, the condition

$$\mathbf{A}_{\tau\tau} + \mathbf{A}_{\sigma\sigma} = 0 \tag{4.26}$$

implies

$$\kappa^2 = v_y^2 + \lambda_1(\omega_1^2 + n_1^2) + \lambda_2(\omega_2^2 + n_2^2) \tag{4.27}$$

and hence

$$\begin{aligned}
E^2 &= P_y^2 + \frac{1}{\lambda_1}(P_{\phi_1} + P_{\psi_1})^2 + \lambda_1(2\pi\tau_1\Pi n_1)^2 + \\
&+ \frac{1}{\lambda_2}(P_{\phi_2} + P_{\psi_2})^2 + \lambda_2(2\pi\tau_1\Pi n_2)^2 \ .
\end{aligned} \tag{4.28}$$

The relation given above has similar form as the relation that was derived in previous section for special case $P_r = 0$. Since Π determines the number of fundamental strings that

propagate on the world-volume of D1-brane we can interpret the solution given above as a bound state of Π fundamental string and one D1-brane that is localized in radial direction. It is a great challenge to find corresponding interpretation of this configuration in dual holographic theory.

Acknowledgement

This work was supported by the Czech Ministry of Education under Contract No. MSM 0021622409.

References

[1] M. B. Green, J. A. Harvey and G. W. Moore, *"I-brane inflow and anomalous couplings on D-branes,"* Class. Quant. Grav. **14** (1997) 47 [arXiv:hep-th/9605033].

[2] N. Itzhaki, D. Kutasov and N. Seiberg, *"I-brane dynamics,"* JHEP **0601** (2006) 119 [arXiv:hep-th/0508025].

[3] L. Y. Hung, *"Comments on I1-branes,"* JHEP **0705** (2007) 076 [arXiv:hep-th/0612207].

[4] M. Berg, O. Hohm and H. Samtleben, *"Holography of D-brane reconnection,"* JHEP **0704** (2007) 013 [arXiv:hep-th/0612201].

[5] L. Y. Hung, *"Intersecting D7-branes, I5-branes and conifolds,"* JHEP **0703** (2007) 071 [arXiv:hep-th/0612070].

[6] L. Grisa, *"Delocalization from anomaly inflow and intersecting brane dynamics,"* JHEP **0703** (2007) 017 [arXiv:hep-th/0611331].

[7] E. Antonyan, J. A. Harvey and D. Kutasov, *"Chiral symmetry breaking from intersecting D-branes,"* Nucl. Phys. B **784** (2007) 1 [arXiv:hep-th/0608177].

[8] E. Antonyan, J. A. Harvey and D. Kutasov, *"The Gross-Neveu model from string theory,"* Nucl. Phys. B **776** (2007) 93 [arXiv:hep-th/0608149].

[9] E. Antonyan, J. A. Harvey, S. Jensen and D. Kutasov, *"NJL and QCD from string theory,"* arXiv:hep-th/0604017.

[10] J. Kluson, *"Dynamics of probe brane in the background of intersecting fivebranes,"* Phys. Rev. D **73** (2006) 106008 [arXiv:hep-th/0601229].

[11] J. Kluson, *"Note about static D1-brane in I-brane background,"* JHEP **0601** (2006) 045 [arXiv:hep-th/0511304].

[12] J. Kluson, *"Dynamics of D1-brane in I-brane background,"* JHEP **0512** (2005) 016 [arXiv:hep-th/0510243].

[13] A. A. Tseytlin, *"Spinning strings and AdS/CFT duality,"* arXiv:hep-th/0311139.

[14] A. A. Tseytlin, *"Semiclassical strings and AdS/CFT,"* arXiv:hep-th/0409296.

[15] J. Plefka, *"Spinning strings and integrable spin chains in the AdS/CFT correspondence,"* arXiv:hep-th/0507136.

[16] D. Kutasov, *"Accelerating branes and the string / black hole transition,"* arXiv:hep-th/0509170.

[17] S. Thomas and J. Ward, *"Geometrical tachyon kinks and NS5 branes,"* JHEP **0510** (2005) 098 [arXiv:hep-th/0502228].

[18] J. Kluson, *“Note about non-BPS and BPS Dp-branes in near horizon region of N Dk-branes,”* JHEP **0503** (2005) 071 [arXiv:hep-th/0502079].

[19] S. Thomas and J. Ward, *“D-brane dynamics near compactified NS5-branes,”* JHEP **0506** (2005) 062 [arXiv:hep-th/0501192].

[20] B. Chen and B. Sun, *“Note on DBI dynamics of Dbrane near NS5-branes,”* Phys. Rev. D **72** (2005) 046005 [arXiv:hep-th/0501176].

[21] J. Kluson, *“Non-BPS Dp-brane in Dk-brane background,”* JHEP **0503** (2005) 044 [arXiv:hep-th/0501010].

[22] Y. Nakayama, K. L. Panigrahi, S. J. Rey and H. Takayanagi, JHEP **0501** (2005) 052 [arXiv:hep-th/0412038].

[23] S. Thomas and J. Ward, *“D-brane dynamics and NS5 rings,”* JHEP **0502** (2005) 015 [arXiv:hep-th/0411130].

[24] D. Bak, S. J. Rey and H. U. Yee, *“Exactly soluble dynamics of (p,q) string near macroscopic fundamental strings,”* JHEP **0412** (2004) 008 [arXiv:hep-th/0411099].

[25] J. Kluson, *“Non-BPS Dp-brane in the background of NS5-branes on transverse R**3 xS**1,”* JHEP **0503** (2005) 032 [arXiv:hep-th/0411014].

[26] J. Kluson, *“Non-BPS D-brane near NS5-branes,”* JHEP **0411** (2004) 013 [arXiv:hep-th/0409298].

[27] D. Kutasov, *“A geometric interpretation of the open string tachyon,”* arXiv:hep-th/0408073.

[28] D. A. Sahakyan, *“Comments on D-brane dynamics near NS5-branes,”* JHEP **0410** (2004) 008 [arXiv:hep-th/0408070].

[29] A. Ghodsi and A. E. Mosaffa, *“D-brane dynamics in RR deformation of NS5-branes background and tachyon cosmology,”* Nucl. Phys. B **714** (2005) 30 [arXiv:hep-th/0408015].

[30] K. L. Panigrahi, *“D-brane dynamics in Dp-brane background,”* Phys. Lett. B **601** (2004) 64 [arXiv:hep-th/0407134].

[31] Y. Nakayama, Y. Sugawara and H. Takayanagi, *“Boundary states for the rolling D-branes in NS5 background,”* JHEP **0407** (2004) 020 [arXiv:hep-th/0406173].

[32] C. P. Burgess, N. E. Grandi, F. Quevedo and R. Rabadan, *“D-brane chemistry,”* JHEP **0401** (2004) 067 [arXiv:hep-th/0310010].

[33] Y. Nakayama, *“Black Hole - String Transition and Rolling D-brane,”* arXiv:hep-th/0702221.

[34] D. M. Hofman and J. M. Maldacena, *“Giant magnons,”* J. Phys. A **39** (2006) 13095 [arXiv:hep-th/0604135].

[35] R. Ishizeki and M. Kruczenski, *“Single spike solutions for strings on S2 and S3,”* arXiv:0705.2429 [hep-th].

[36] N. P. Bobev and R. C. Rashkov, *“Spiky Strings, Giant Magnons and beta-deformations,”* Phys. Rev. D **76** (2007) 046008 [arXiv:0706.0442 [hep-th]].

[37] C. S. Chu, G. Georgiou and V. V. Khoze, *“Magnons, classical strings and beta-deformations,”* JHEP **0611** (2006) 093 [arXiv:hep-th/0606220].